NAME (underline family name):

STUDENT NUMBER: **SIGNATURE:**

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 263

ORDINARY DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

Examiner: J. Hurtubise
Associate Examiner: K.P. Russell
Date: Tuesday, April 25, 2007
Time:

Instructions

1. Write your name and student number on this examination script.
2. No books, calculators or notes allowed.
3. This examination booklet consists of this cover, 10 pages of questions and 2 blank pages (cover + 12 numbered pages in all). Please take a couple of minutes in the beginning of the examination to scan the problems. (Please inform the invigilator if the booklet is defective.)
4. Answer all questions. You are expected to show all your work. All solutions are to be written on the page where the question is printed. You may continue your solutions on the facing page. If that space is exhausted you may continue on the blank pages at the end, clearly indicating any continuation on the page where the question is printed.
5. Circle your answers where confusion could arise.

GOOD LUCK!

Score Table

Problem	Points	Out of
1.		10
2.		10
3.		10
4.		10
5.		10
6.		10
7.		10
8.		10
9.		10
10.		10
Total:		100

1. (10 marks) Solve

$$y' = \frac{x^2 + 3y^2}{2xy}, \quad x > 1.$$

with the initial condition $y(2) = 6$.

2. (10 marks) Solve the initial value problem

$$x(2+x)y' + 2(1+x)y = 1 + 3x^2, \quad y(-1) = -1$$

3. (10 marks) Find the appropriate integrating factor of the form $x^\alpha y^\beta$ for the equation

$$(3 - 20x^2y)dx + (2xy^{-1} - 12x^3)dy = 0$$

Solve the equation for the initial conditions $y(1) = 2$.

4. (10 marks) Find the general solution to

$$y^{iv} - 6y''' + 9y'' = x$$

5. (10 marks) Find the general solution of

$$y'' - 2y' + y = e^x/(1 + x^2)$$

6. (10 marks) Using Laplace transforms and the following table, solve the initial value problem

$$y'' + y = u(t - \pi/2)e^t + \delta(t - \pi), \quad y(0) = y'(0) = 0$$

function $f(t)$	Laplace transform $F(s)$
1	$1/s \quad (s > 0)$
t^n	$n!/s^{n+1} \quad (s > 0)$
e^{at}	$1/(s-a) \quad (s > a)$
$\sin at$	$a/(s^2 + a^2) \quad (s > 0)$
$\cos at$	$s/(s^2 + a^2) \quad (s > 0)$
$e^{-at} f(t)$	$F(s + a)$
$u(t-a)$ or $u_a(t)$ $(a \geq 0)$	$e^{-as}/s \quad (s > 0)$
$\delta(t - a) \quad (a > 0)$	e^{-as}
$u(t-a)f(t-a)$ or $u_a(t)f(t-a)$	$e^{-as}F(s)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) \ldots - f^{(n-1)}(0)$
$f * g(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$

7. (10 marks)
The 5×5 matrix

$$\begin{pmatrix} 1 & 2 & 5 & 1 & -5 \\ 1 & 2 & 6 & 3 & -7 \\ -1 & -2 & -5 & 0 & 6 \\ 3 & 6 & 14 & 2 & -12 \\ 1 & 2 & 1 & -7 & 3 \end{pmatrix}$$

row reduces to

$$\begin{pmatrix} 1 & 2 & 5 & 1 & -5 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Compute bases of the image and the kernel of M.

8. (10 marks) Apply the Gram-Schmidt method to compute an orthonormal basis of the vector space spanned by

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1 \\ -3 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} -1 \\ 1 \\ -5 \\ 3 \end{pmatrix}$$

9. (10 marks) Find the eigenvalues and corresponding eigenvectors of the symmetric matrix

$$A = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$$

Find an orthogonal matrix P and a diagonal matrix D with $A = PDP^T$.

Compute the matrices in the standard basis of orthogonal projection onto each eigenspace.

10. (10 marks)

Find the general solution to the linear system

$$Y' = \begin{pmatrix} 4 & -5 & 0 \\ 5 & -4 & 0 \\ 0 & 0 & -2 \end{pmatrix} \cdot Y$$

Express your basis of solutions in real form (i.e no complex exponentials). Which solutions decay to 0 as x goes to $+\infty$? Solve for the initial conditions $Y(0) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$.

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